Population Projection of Nepal: A Logistic Approach

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Abstract: *Population growth is a dynamic process which depends upon many variables which result the population projection as a very complicated task. In this article, we try to project the population of Nepal for upcoming 100 years and also project the trend of population growth for next 300 years. In this projection, we use the Logistic Growth Model, a more realistic model of population projection. Here we use the every 10 years data of census and also calculate all the intermediate year's data by using the exponential growth model. Thus, we use all together 41 years data in our calculation. By using the least square method to fit the Logistic Model in the past population and using the MATLAB, we calculate the logistic growth rate of population of Nepal is* $r = 3.6955\%$*. The carrying capacity of Nepal is* $K = P_{max} = 4,38,14,550$ *and the inflection year at which the population is half of the maximum population (*$\frac{K}{2} = 2,19,07,275$*) of Nepal as 1999 from when the population of Nepal will start to be more stable. The population growth rate will be remarkably reduced around the 2071 and the population of Nepal will start to remain more stable from around 2100. As the area of Nepal, its natural resources and possibilities of the dynamical connectivity between the rapid economically growing neighbors, the future population of Nepal seems to be manageable.*
Key Words: Nepal, Logistic Model, population projection

1 Introduction

Population growth refers to the change in the number of persons residing in an area during a specified period of time. The change may be either positive or negative. Human population is multidisciplinary in nature, which provides enlightening insights into changes in population structure and behavior, pinpointing causal relationships and refining explanations [1]. Due to the limited resources of the earth, the world has been facing the threaten of the future of human beings due to the non managing population growth. The population growth determines the demography of a country and hence it has important role for future plans to be adopted by the country. The population growth is a dynamical process depends upon the growth rate. The growth rate itself depends upon the fertility, mortality rates and net migration. These factors are also associated with so many other variables. Over the last 2000 years, the annual rate of increase of global population grew about 50-fold from an average of 0.04% per year between AD. 1 and 1650 to its all-time peak of 2.1 % per year around 1965 to 1970 . The growth rate has since declined haltingly to about 1.6% per year[4]. During the twentieth century, as developments in health care and food production have raised life expectancy and improved standards of nutrition, the world's population growth has been greater than at any other time in its history. Although overall population growth rates are estimated by the United Nations (UN) to have peaked in the 1965-1970 period [9].

The history of census taking in Nepal dates back to 1911 when the first census was carried out all over the country with the purpose of collecting data on youth. The head counting covered adult males possibly to estimate the military strength of the nation during the regime of Bhimsen Thapa (1806-1838). The population of Nepal in 1911 was 5,638,749 and in 2011 it becomes 26,494,504. So, in the period of 100 years Nepal's population rapidly raised by nearly five folds. The population growth rate of Nepal till the 1930 was negative and in 1941-1942 it was 1.16 and then with many fluctuations during the past periods now in 2011 it is 1.35 [2]. It is not worth to think about the fluctuations of growth rates in future also. The fluctuation of growth rate results the projection of population as a very complicated task.

United Nations in its report "World population to 2300" projected the world's population up to 2300 along with some components of population growth including the projections of some regions [8]. This report did not include the country wise projections. World Population Review (2017) provides the projection of population of the world along with the country wise projection including the projections of some components of population growth like fertility, mortality, net migration etc. up to 2095 [6]. CBS in its report "National Population and Housing Census 2011 (Population Projection 2011-2031)" projects the population of Nepal for 2011-2030 [3]. These two projections somehow give the similar approximation about Nepal's future population but these projections do not describe about the used modeling of the projections.

A population model is a type of mathematical model that is applied to the study of population dynamics. There are many mathematical models for the projection of population. A simple exponential model gives sufficient approximation in estimation of population but does not defines a saturation point. Hence the population estimated increases exponentially without any upper limit giving an unrealistic figure for longer time period. This is because it does not considers the environmental factors and hence suitable for very short period of time. Logistic model tells that the population growth rate decreases as the population reaches the carrying capacity (limiting value of the population or maximum supportable population) or saturation point of the environment. The Logistic Model is more accurate than the exponential model [7].

Human population trends are centrally important to environmental science because they help to determine the environmental impact of human activities. Rising populations put increasing demands on natural resources such as land, water, and energy supplies. As human communities use more resources, they generate contaminants, such as air and water pollution and greenhouse gas emissions, along with increasing quantities of waste. Good understanding of contemporary demographic structure and population dynamics underpins effective planning and decision making for the future. In this article, we use the logistic Growth Model to estimate the population of Nepal for next 100 years and predicts the trends of the population for next 300 years.

2 Methodology

For this paper, secondary data were collected from the various sources mainly from CBS, world population prospect and United Nation Population Fund. MATLAB software is used to compute the predicted population values and in plotting down the graphs of actual and predicted population values against time in years. We also use least square method to compute the population growth rate, the carrying capacity and the year when the population of Nepal will be approximately a half of the value of its carrying capacity.

2.1 Development of the model

Thomas R. Malthus [5], in 1798 purposed the mathematical model $P(t) = P_0 e^{(\alpha t)}$ where P_0 is the initial population and P is the population after time t years called Malthusian law of population growth or Exponential law of population growth and is widely regarded in the field of population ecology as the first principle of population dynamics. This model assumes that the population grows uncontrollably in future with its growth rate. But in practice it will not happen due to the various environmental as well as social factor. So there will need a correction factor for this model . A Belgian Mathematician Verhulst [10] showed that the population growth not only depends on the population size but also on how far this size is from its upper limit i.e. its carrying capacity (maximum supportable population). He modified Malthus's Model to make the population size proportional to both the previous population and a new term $\dfrac{\alpha - \beta P(t)}{\alpha}$, where α and β are called the vital coefficients of the population. So the modified equation using this new term is:

$$\frac{dP(t)}{dt} = \frac{\alpha P(t)(\alpha - \beta P(t))}{\alpha} \tag{1}$$

This is a nonlinear differential equation and known as the logistic law of population growth. Solving this we get

$$P = \frac{\frac{\alpha}{\beta}}{1 + (\frac{\frac{\alpha}{\beta}}{P_0} - 1)\exp(-\alpha t)} \tag{2}$$

Where P_0 is the population when $t = 0$. For more details of solution see [11]. If we take $t \to \infty$ we get $P_{max} = \dfrac{\alpha}{\beta}$ (since $\alpha > 0$) . This is the maximum supportable population of the region. We called it as carrying capacity and denoted by K . Now we determine the value of α, β and K by using the least square method. Denote $C = (\dfrac{\frac{\alpha}{\beta}}{P_0} - 1)$

in equation (2) and differentiate twice we get

$$\frac{d^2P}{dt^2} = \frac{C\alpha^3 \exp(\alpha t)(C - \exp(\alpha t))}{\beta(C + \exp(\alpha t))^3} \tag{3}$$

solving by putting $\frac{d^2P}{dt^2} = 0$, we get $C = \exp(\alpha t)$ and hence we get, $t = \frac{\log C}{\alpha}$. This gives the time for the point of inflection. When we put this value in the above equation we get, $P = \frac{\alpha}{2\beta} = \frac{K}{2}$, i.e the time of inflection is that time when the population is half of the carrying capacity. Let the time when the point of inflection occurs be $t = t_k$. Then $C = \exp(\alpha t)$ becomes $C = \exp(\alpha t_k)$. Using this new value of C and replacing $\frac{\alpha}{\beta}$ by K from eq (2) we get

$$P = \frac{K}{1 + \exp(-\alpha(t - t_k))} \tag{4}$$

Let p be the actual population and P be the corresponding projected population in a certain year t, then the error of the estimation is $(P - p)$. Thus, the total squared error, e, in fitting the curve is given by

$$e = \sum_{i=1}^{n} (P_i - p_i)^2 \tag{5}$$

This equation contains the the three parameters α, K and t_k. Let us put $P = Kh$ where

$$h = \frac{1}{1 + \exp(-\alpha(t - t_k))}$$

Putting the value of $P = Kh$ in equation (5) and using the properties of inner product we get

$$\begin{aligned}
e &= \sum_{i=1}^{n} (P_i - p_i)^2 \\
&= (P_1 - p_1)^2 + (P_2 - p_2)^2 + ... + (P_n - p_n)^2 \\
&= (Kh_1 - p_1)^2 + (Kh_2 - p_2)^2 + ... + (Kh_n - p_n)^2 \\
&= |Kh_1 - p_1, Kh_2 - p_2, ..., Kh_n - p_n|^2 \\
&= |(Kh_1, Kh_2, ..., Kh_n) - (p_1, p_2, ..., p_n)|^2 \\
&= |KH - W|^2 \\
&=< KH - W, KH - W > \\
&= K^2 < H, H >^2 - 2K < H, W > + < W, W >
\end{aligned}$$

where $H = (h_1, h_2, ..., h_n)$ and $W = (p_1, p_2, ..., p_n)$. Taking partial derivative of e with respect to K and equating it to zero, we obtain $2K < H, H > -2 < H, W >= 0$. This gives

$$K = \frac{< H, W >}{< H, H >} \tag{6}$$

Substituting this value in above equation we get

$$e =< W, W > - \frac{< H, W >^2}{< H, H >} \tag{7}$$

This equation is converted into an error function, MATLAB program (code are given in appendix) we calculate the growth rate α, inflection year (t_k) and carrying capacity (K). By using all these values in the equation (2) we estimates the past population and make a prediction about the future population of Nepal for next 100 years.

3 Results

3.1 Approximation of the population of Nepal by Logistic Growth Model

The history of the census of Nepal is from 1911 and Nepal celebrated the century of census in 2011 which is the latest census of Nepal. Here we use the data only from 1971 because before then, we have no appropriate data to fit the logistic model. As we seen that the negative growth rate from 1911-1930 and the growth rate also fluctuated from 1930-1961. The logistic model does not work in the negative growth rate and assumes the exponential growth rate in the initial time and it becomes more stable as it reaches closer to the carrying capacity, so that's why we take the data only from 1971 . Here we use the every 10 years data of census and also calculate all the intermediate years data by using the exponential growth model. Thus we use all together 41 years data in our calculation. By using the least square method to fit the Logistic Model in the past population and using the MATLAB, we calculate the logistic growth rate of population $r = 3.6955\%$. The carrying capacity of Nepal is $K = P_{max} = 4,38,14,550$ and the inflection year at which the population is half of the maximum population $(\frac{K}{2} = 2,19,07,275)$ of Nepal as 1999 from when the population of Nepal will start to be more stable. We let $t = 0$ for the year 1971 and using all these values in eq (2), we finally derive the the following equation for the approximation and prediction of the population

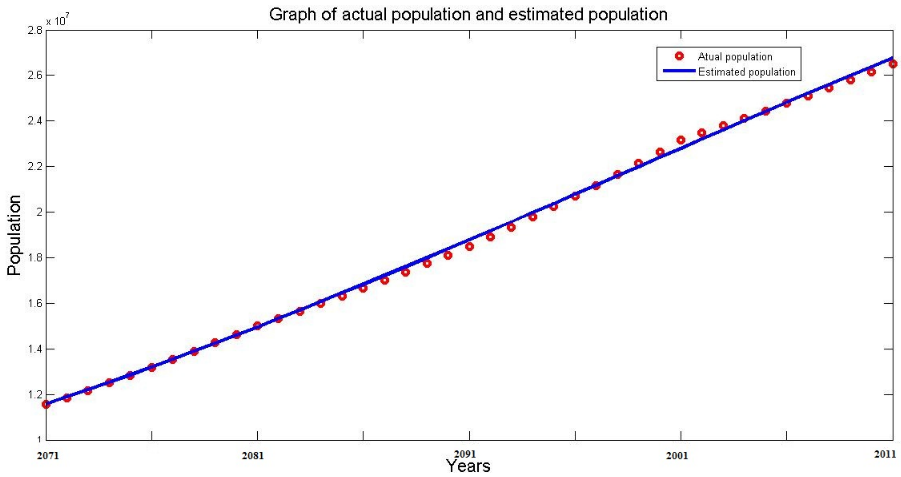
$$P = \frac{43814550}{1 + (2.7915035008) \times (0.96371502)^t} \tag{8}$$

The following table shows the estimated population by using the model and their corresponding actual population values.

Table 1: Actual population and approximated population by the model

Year	Actual Pop.	Estimated Pop.	**Differences**	Year	Actual Pop.	Estimated Pop.	**Differences**
1971	1,15,55,983	1,15,55,983	0	1981	1,50,22,839	1,49,59,053	63,786
1991	1,84,91,097	1,87,80,826	2,89,729	2001	2,31,51,423	2,28,07,257	3,44,166
2011	2,64,94,504	2,67,73,689	2,79,185				

From Table 1, we see that the actual data and predicted values are very close to each other.The maximum error is shown in the data of 2001 but it is only about 1.5%. This indicates that the error between them is very small.
The following figure shows the graphical representation of the actual and predicted population.



From this fig., we can see that the actual data points and predicted values are very close to one another. This indicates that the error between them is very small and hence the logistic growth model is appropriate for the past population of Nepal.

3.2 Projection of the population of Nepal for next century

The modeling of estimation of the population is a very complicated work because there are so many variables associated with it in the past and also the new variables can be associated in future.The logistic growth model is used here and by using the equation (8) derived from this model, we predict the population of Nepal from 2021 to 2111 for this century in every interval of 10 years shown by the following table.

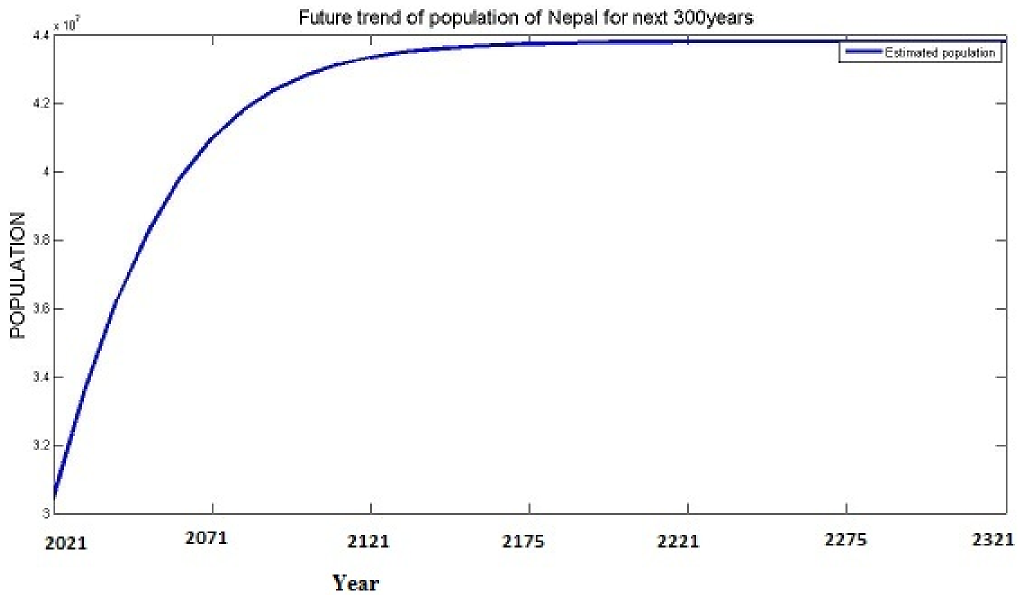
Table 2: Projection of the population of Nepal for next century

Year	Projected Population	Year	Projected Population
2021	3,04,30,690	2031	3,36,02,238
2041	3,62,10,041	2051	3,82,61,959
2061	3,98,21,269	2071	4,09,75,181
2081	4,18,12,418	2091	4,24,11,237
2101	4,28,35,150	2111	4,31,33,064

The above table shows that, the population growth up to 2051 is quite high but later on the population growth is lowering and we find a remarkable low growth from 2101 to 2111 in which only about 3 lakhas people assume to be added to the previous population shows the almost saturation level of the logistic model.

3.3 Future trends of Population growth of Nepal

By using the limited data, the prediction about the trends may not be more appropriate. We have limited data and also it is not known as international standard. The 1961 census is generally accepted as the first scientific census in terms of international standards and comparisons. The subsequent censuses were all conducted at one point of time and adhere to international standards [2]. The following figure shows the trend of population growth form 2021 to 2321 i.e for next 300 years.



The above graph shows that the population growth rate will be remarkably reduced around the 2071 and the population of Nepal will be nearly in its saturation level from around 2100. This is the theoretical projection but in practical it will not worth to think about the crossing of population than its carrying capacity. At this situation we will have the negative growth rates. So, in the long time the population may be fluctuate but go to asymptotically with its carrying capacity.

4 Discussion

The logistic growth rate of population of Nepal is $r = 3.6955\%$. The carrying capacity (limiting value of the population or maximum supportable population) of Nepal is $K = P_{max} = 4,38,14,550$ and the inflection year at which the population is half of the maximum population ($\frac{K}{2} = 2,19,07,275$) of Nepal as 1999 from when the population of Nepal will start to be more stable. A significantly low growth rate is seen around the 2071's and approximately the saturation period is seen around 2100's in this model. A report [8] by Population Division of the Department of Economic and Social Affairs of the United Nations indicates that in the medium scenario, world population rises from 6.1 billion persons in 2000 to a maximum of 9.2 billion persons in 2075 which is consistence with our projection as in case of Nepal. But this report indicates that the population of the world declines thereafter to reach 8.3 billion in 2175. Another projection [6] indicates the maximum population of Nepal will be 36,416,318 in 2055 after then it will start to decrease. In 2055 our approximation gives the population 3,89,40,156 but in our projection the negative growth never occur due the applied model. The Central Bureau of Statistics(CBS) Nepal in its report [3] projected the population of Nepal in three categories High, Medium and Low. The medium projection of its quite resembles to our projection can be shown in the following table:

Table 3: Projection of the population of Nepal by CBS and Logistic Method

Year	Projection by CBS	Projection by Logistic Model	Differences
2011	2,64,94,504	2,67,73,689	2,79,185
2016	2,84,31,494	2,86,54,074	2,22,580
2021	3,03,78,055	3,04,30690	52,635
2026	3,21,44,921	3,20,84,339	-60,582
2031	3,35,97,032	3,36,02,238	5,202

The over all estimation of the past population of Nepal by this model is somehow fitted with the actual data because we have no differences more than by 1.5% and also we estimate more than the actual data. This is quite good approximation also supported by the data of post enumeration survey which indicates that the net omission is estimated to be 5.3$\%$ for 2001 and 3.6$\%$ for 2011[2]. The above table also shows that the future prediction is also consistence with CBS.

5 Conclusion

The analysis of Nepal's demographic data over the last several decades indicates that it has not only already experienced the onset of its demographic transition, but that in fact it is experiencing rapid demographic change as a result of positive socioeconomic developments. Within a few decades, Nepal has achieved significant declines in mortality rates, fertility rates, and population growth rates, while experiencing rapid improvements in life expectancy. In this article, we project the population of Nepal for coming 100 years and also project the trend for next 300 years by Logistic Model. This projection indicates that the population of Nepal will not more than 4,38,14,550 and reach its saturation limits almost in next 50 years. The human population, whether the constant growth rate can be observed is in doubt. This is mainly because we can hardly define the conditions. Human reproduce sexually and have consciousness. We cannot be sure under what condition will an individual be reproduced. Therefore, it is in doubt whether models based on constant growth rate can explain human population growth. Taking carrying capacity into regards, logistic growth model improves the preceding exponential growth model. However, whether it can describe human population growth is in dispute. The size of the population has significant role in the formation of the various plans for the development of the nation. Without the long term plan for the development of the nation, the desired need of the people would not be in realms. In each and every developmental plan, the size of the population should be in account. That is why the projection of population is significantly important. But we should keep in mind that human influence on the planet has increased faster than the human population. So we should aware about the use of resources of planet. As the area of Nepal, its natural resources and possibilities of the dynamical connectivity between the rapid economically growing neighbors China and India, the future population of Nepal seems to be manageable.

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Appendix

MATLAB Code for the Model

```
%This Progam Estimates the Population Parameters using Least Square Approximations
function Population
clc;close all; clear all;
global t p H
x=fminsearch(@(x) SquareError(x), [0.1 2100]);
% x is the vector with two components x(1)=r and x(2)=tk.
r=x(1);
tk=x(2);
K=(H'*p)/(H'*H);
% Results will be printed in Command Window
fprintf('\ t \ t \ t \ t Results \ n',r);
fprintf('——————————————\ n',r);
fprintf('The Growth Rate is r = %5.6f \ n',r);
fprintf('The Inflection time tk = %d Years \ n',round(tk));
fprintf('Carrying Capicity is K = % d \ n',round(K));
fprintf('——————————————\ n',r);
```

```
%Estimated Population
t=0:10:40;
P = 115983531./(1 + (17.4589909) * (0.97602215).^t);
end
function e=SquareError(x)
global t p H
% x = [x(1), x(2)] and x(1)=r and x(2)=tk
t=0:1:40;
p=[11555983,11863188,12178559,12502315,...
12834677,13175875,13526143,13885722,14254861,14633813,15022839,...
15338153,15660086,15988776,16324364,16666997,17016820,17373986,...
17738649,18110966,18491097,18911422,19341301,19780952,20230596,...
20690462,21160781,21641790,22133734,22636860,23151423,23465807,...
23784461,24107441,24434808,24766620,25102938,25443823,25789337,...
26139543,26494504];
H=1./(1+exp(-x(1)*(t-x(2))));
p=p';
H=H';
e = (p'*p) - ((H'*p)^2/(H'*H));
end
```