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On the Structure of Complex Variable Struve and Modified Struve Function of the Second Kind

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Abstract: *In this note we derive explicit expressions for real and imaginary parts of the Struve function $\mathbf{K}_\nu(z)$ of the second kind and also the modified Struve function $\mathbf{M}_\nu(z)$ of the second kind of real parameter ν and complex variable z. The derivation method is based on trigonometric–type differentiation operators.*

Keywords: Bessel and modified Bessel functions of the first kind, Struve functions, Modified Struve functions

1 Introduction and Preparation

A recent work concerns the determination of the steady periodic average concentration of an electroactive component over the surface of a disk electrode embedded in an insulator comprising the horizontal plane, and which is subjected to a periodic prescribed flux, and which is reported in [1–3]. The mathematics of this study is based on the solutions of non–homogeneous modified Bessel differential equation, called Struve differential equation, that is, to extract the real and imaginary parts of the modified Struve function of the second kind $\mathbf{M}_1(z)$, when the parameter ν is real, and the variable z is complex.

In the article [4] the authors proved that a special form Feynman integral is expressible in different forms involving real and imaginary parts of the Gauss hypergeometric function ${}_2F_1$ having real parameters and complex variable as well as generalized hypergeometric ${}_2F_2$ and ${}_3F_2$, Horn H_4 and Appell F_2 functions. The results turn out to be either closed form expressions and/or computable functional series built from hypergeometric terms.

Motivated by the previous considerations in this short note we present the real and imaginary parts of Struve functions of the second kind $\mathbf{K}_\nu(z)$ and modified Struve function of the second kind $\mathbf{M}_\nu(z)$ both for real order parameter ν and complex variable z. In doing so we will need the definitions and integral expressions of the involved Bessel and Struve functions.

The Struve function and the modified Struve function, both of the first kind of the order ν have the power series expansions

$$\mathbf{H}_\nu(z) = \left(\frac{z}{2}\right)^{\nu+1} \sum_{n\geq 0} \frac{(-1)^n \left(\frac{z}{2}\right)^{2n}}{\Gamma(n+\frac{3}{2})\Gamma(n+\nu+\frac{3}{2})}\,,$$

$$\mathbf{L}_\nu(z) = \left(\frac{z}{2}\right)^{\nu+1} \sum_{n\geq 0} \frac{(-1)^n \left(\frac{z}{2}\right)^{2n}}{\Gamma(n+\frac{3}{2})\Gamma(n+\nu+\frac{3}{2})}\,,$$

respectively. Their principal values coincide with the principal values of $z^{\nu+1}$, whilst both sums are entire functions of z and ν, consult [5, p. 288].

The Struve function and the modified Struve function of the second kind of the order ν can be expressed as [5, p. 288, Eqs. 11.2.5-6]

$$\mathbf{K}_\nu(z) = \mathbf{H}_\nu(z) - Y_\nu(z)\,, \tag{1}$$

$$\mathbf{M}_\nu(z) = \mathbf{L}_\nu(z) - I_\nu(z)\,, \tag{2}$$

where the Bessel function of the second kind (expressed by virtue of the Bessel function of the first kind) [5, p. 217]

$$Y_\nu(z) = \cot(\nu\pi) J_\nu(z) - \csc(\nu\pi) J_{-\nu}(x); \qquad J_\nu(z) = \Big(\frac{z}{2}\Big)^\nu \sum_{n\geq 0} \frac{(-1)^n \left(\frac{z}{2}\right)^{2n}}{\Gamma(n+\nu+1)\, n!}\,,$$

and the modified Bessel function of the first kind [5, p. 249, Eq. 10.25.2]

$$I_\nu(z) = \Big(\frac{z}{2}\Big)^\nu \sum_{n\geq 0} \frac{\left(\frac{z}{2}\right)^{2n}}{\Gamma(n+\nu+1)\, n!}$$

occur.

Finally, the Laplace transform integral representation of the Struve function of the second kind is [5, p. 292, Eq. 11.5.2]

$$\mathbf{K}_\nu(z) = \frac{z^\nu}{2^{\nu-1}\sqrt{\pi}\,\Gamma(\nu+\frac12)} \int_0^\infty \mathrm{e}^{-zt}(1+t^2)^{\nu-\frac12}\mathrm{d}t, \qquad \Re(z)>0, \tag{3}$$

and the same for $\mathbf{M}_\nu : (0,\infty) \to \mathbb{R}$, defined by (2) has the integral representation [5, p. 292, Eq. 11.5.4]

$$\mathbf{M}_\nu(z) = -\frac{z^\nu}{2^{\nu-1}\sqrt{\pi}\,\Gamma(\nu+\frac12)} \int_0^1 \mathrm{e}^{-zt}(1-t^2)^{\nu-\frac12}\mathrm{d}t, \qquad \Re(\nu) > -\frac12. \tag{4}$$

2 Auxiliary Lemmata

Firstly, we introduce the differential operators defined by the Maclaurin series of the sine and cosine functions, *viz.*

$$\sin(D) = \sum_{n\geq 0} \frac{(-1)^n\, D^{2n+1}}{(2n+1)!}\,, \qquad \cos(D) = \sum_{n\geq 0} \frac{(-1)^n\, D^{2n}}{(2n)!}\,. \tag{5}$$

Here $D = \frac{\mathrm{d}}{\mathrm{d}x}$ stands for the input differential operator which acts on a suitable function $f \in \mathrm{C}^\infty(\mathbb{R})$. We denote throughout the output of applications of these operators as $\sin(D)[f], \cos(D)[f]$ and $\mathrm{e}^{\mathrm{i}D}[f]$.
Next, we report on two auxiliary integrals of Laplace transform type.

Lemma 1. *For all $\Re(\nu) > -\frac12$ and all $x>0, y\in\mathbb{R}$ we have*

$$\mathscr{I}_c^\nu(x,y) := \int_0^\infty \mathrm{e}^{-xt}\cos(yt)\big(1+t^2\big)^{\nu-\frac12}\,\mathrm{d}t = 2^{\nu-1}\sqrt{\pi}\,\Gamma\big(\nu+\tfrac12\big)\cos(yD)\Big[\frac{\mathbf{K}_\nu(x)}{x^\nu}\Big], \tag{6}$$

$$\mathscr{I}_s^\nu(x,y) := \int_0^\infty \mathrm{e}^{-xt}\sin(yt)\big(1+t^2\big)^{\nu-\frac12}\,\mathrm{d}t = -2^{\nu-1}\sqrt{\pi}\,\Gamma\big(\nu+\tfrac12\big)\sin(yD)\Big[\frac{\mathbf{K}_\nu(x)}{x^\nu}\Big]. \tag{7}$$

Proof. Expanding the cosine into Maclaurin series for some fixed $y\in\mathbb{R}$, by the legitimate change of the order of summation and integration we conclude

$$\begin{aligned}\mathscr{I}_c^\nu(x,y) &= \sum_{n\geq 0}\frac{(-y^2)^n}{(2n)!} D^{2n}\int_0^\infty \mathrm{e}^{-xt}\big(1+t^2\big)^{\nu-\frac12}\,\mathrm{d}t \\ &= 2^{\nu-1}\sqrt{\pi}\,\Gamma\big(\nu+\tfrac12\big)\sum_{n\geq 0}\frac{(-y^2)^n}{(2n)!}\, D^{2n}\Big[\frac{\mathbf{K}_\nu(u)}{u^\nu}\Big]\Big|_{u=x}\,,\end{aligned}$$

where (3), and then (1) are used. This confirms (6). We prove (7) by a similar procedure applied to the sine Maclaurin series. □

The following pair of integrals are the Laplace transforms of the functions

$$\cos(yt)\big(1-t^2\big)^{\nu-\frac12}\chi_{[0,1]}(t) \quad \text{and} \quad \sin(yt)\big(1-t^2\big)^{\nu-\frac12}\chi_{[0,1]}(t),$$

respectively; here $\chi_S(t)$ stands for the characteristic function of the set S.

Lemma 2. *For all $\Re(\nu) > -\frac{1}{2}$ and all $x, y \in \mathbb{R}$ we have*

$$\mathscr{I}_c^{\,\nu}(x,y) := \int_0^1 \mathrm{e}^{-xt}\cos(yt)\big(1-t^2\big)^{\nu-\frac{1}{2}}\,\mathrm{d}t = 2^{\nu-1}\sqrt{\pi}\,\Gamma\big(\nu+\tfrac{1}{2}\big)\cos(yD)\Big[\frac{\mathbf{M}_\nu(x)}{x^\nu}\Big], \tag{8}$$

$$\mathscr{I}_s^{\,\nu}(x,y) := \int_0^1 \mathrm{e}^{-xt}\sin(yt)\big(1-t^2\big)^{\nu-\frac{1}{2}}\,\mathrm{d}t = 2^{\nu-1}\sqrt{\pi}\,\Gamma\big(\nu+\tfrac{1}{2}\big)\sin(yD)\Big[\frac{\mathbf{M}_\nu(x)}{x^\nu}\Big], \tag{9}$$

Proof. Applying the same proving methodology as in the previous Lemma's proof, expand the cosine into Maclaurin series (5) for a fixed $y \in \mathbb{R}$. By virtue of (2) and then using (4) it follows that

$$\begin{aligned}\mathscr{I}_c^{\,\nu}(x,y) &= \sum_{n\geq 0}\frac{(-y^2)^n}{(2n)!}D^{2n}\int_0^1 \mathrm{e}^{-xt}\big(1-t^2\big)^{\nu-\frac{1}{2}}\,\mathrm{d}t \\ &= 2^{\nu-1}\sqrt{\pi}\,\Gamma\big(\nu+\tfrac{1}{2}\big)\sum_{n\geq 0}\frac{(-y^2)^n}{(2n)!}D^{2n}\Big[\frac{I_\nu(x)-\mathbf{L}_\nu(x)}{x^\nu}\Big] \\ &= 2^{\nu-1}\sqrt{\pi}\,\Gamma\big(\nu+\tfrac{1}{2}\big)\sum_{n\geq 0}\frac{(-1)^{n+1}}{(2n)!}(yD)^{2n}\Big[\frac{\mathbf{M}_\nu(x)}{x^\nu}\Big]\,,\end{aligned}$$

to prove (8). The formula (9) is similarly established by the sine Maclaurin series in the integrand. □

3 Main Results

By convention we use from now as usual $z = x + \mathrm{i}y \in \mathbb{C}$ with real and imaginary parts x, y, which comprise the modulus $|z|^2 = x^2 + y^2$ and the argument of z is $\varphi = \arctan(y/x)$. The real and the imaginary parts of the Struve function of the second kind are separately given in closed form expressions.

Generally speaking we consider a function of a complex variable $I(z)$ say, which is defined as the Laplace–Mellin type integral transform of convenient functions g, whilst $h \in \mathrm{L}^1[a,b]; a,b \in \mathbb{R}$, *viz.*

$$I(z) := g(z)\int_a^b \mathrm{e}^{-zt}h(t)\,\mathrm{d}t\,. \tag{10}$$

Then under certain suitable convergence conditions

$$I(z) = g(z)\,\mathrm{e}^{\mathrm{i}D}\left[\frac{I(x)}{g(x)}\right]\,.$$

Indeed, the conditions should allow us to commute infinite sums so, $I(z)$ one transforms as:

$$\begin{aligned}I(z) &= g(z)\int_a^b \mathrm{e}^{-xt}\mathrm{e}^{-\mathrm{i}yt}\,h(t)\,\mathrm{d}t = g(z)\sum_{n\geq 0}\frac{(-\mathrm{i}y)^n}{n!}\int_a^b t^n\mathrm{e}^{-xt}\,h(t)\,\mathrm{d}t \\ &= g(z)\sum_{n\geq 0}\frac{(\mathrm{i}y\,D)^n}{n!}\int_a^b \mathrm{e}^{-xt}\,h(t)\,\mathrm{d}t = g(z)\,\mathrm{e}^{\mathrm{i}D}\left[\frac{I(x)}{g(x)}\right]\,,\end{aligned}$$

having in mind (10). This can be straightforwardly separated into real and imaginary parts. Namely, for a function $f\colon X \mapsto \mathbb{C}; X \subseteq \mathbb{C}$ there exist functions $u, v\colon X \mapsto \mathbb{R}$ so, that $f = u + \mathrm{i}v$. More precisely,

$$u = \frac{1}{2}(f + f^*); \qquad v = \frac{\mathrm{i}}{2}(f^* - f)\,,$$

and the real and the imaginary parts of the input function f are extracted. Here f^* stands for the complex conjugate of certain f. However, direct application of this exponential form does not yield the explicit real and the imaginary parts of the operation *per se*. Therefore, the detailed exposition of our precise extraction results follow below.

Theorem 1. *For all $\Re(\nu) > -\frac{1}{2}$ and all $z \in \mathbb{C}$, for which $\Re(z) = x > 0, \Im(z) = y \in \mathbb{R}$ we have that*

$$\mathbf{K}_\nu(z) = |z|^\nu \left\{ \cos\left(\nu\varphi + yD\Big[\frac{\mathbf{K}_\nu(x)}{x^\nu}\Big]\right) + \mathrm{i}\sin\left(\nu\varphi + yD\Big[\frac{\mathbf{K}_\nu(x)}{x^\nu}\Big]\right)\right\}. \tag{11}$$

Proof. Starting the direct calculations with the Laplace transform (3), and continuing with the results of Lemma 1, we get

$$\begin{aligned}
\mathbf{K}_\nu(z) &= \frac{z^\nu}{2^{\nu-1}\sqrt{\pi}\Gamma(\nu+\frac{1}{2})}\int_0^\infty \mathrm{e}^{-xt}\mathrm{e}^{-\mathrm{i}yt}(1+t^2)^{\nu-\frac{1}{2}}\,\mathrm{d}t \\
&= \frac{|z|^\nu(\cos(\nu\varphi)+\mathrm{i}\sin(\nu\varphi))}{2^{\nu-1}\sqrt{\pi}\Gamma(\nu+\frac{1}{2})}\Big(\mathscr{I}_c^\nu(x,y) - \mathrm{i}\mathscr{I}_s(x,y)\Big) \\
&= \frac{|z|^\nu(\cos(\nu\varphi)+\mathrm{i}\sin(\nu\varphi))}{2^{\nu-1}\sqrt{\pi}\Gamma(\nu+\frac{1}{2})}\Big\{\cos(\nu\varphi)\mathscr{I}_c^\nu(x,y) + \sin(\nu\varphi)\mathscr{I}_s^\nu(x,y) \\
&\quad + \mathrm{i}\Big(\sin(\nu\varphi)\mathscr{I}_c^\nu(x,y) - \cos(\nu\varphi)\mathscr{I}_s^\nu(x,y)\Big)\Big\} \\
&= |z|^\nu\Big\{\cos(\nu\varphi)\cos(yD)\Big[\frac{\mathbf{K}_\nu(x)}{x^\nu}\Big] - \sin(\nu\varphi)\sin(yD)\Big[\frac{\mathbf{K}_\nu(x)}{x^\nu}\Big] \\
&\quad + \mathrm{i}\Big(\sin(\nu\varphi)\cos(yD)\Big[\frac{\mathbf{K}_\nu(x)}{x^\nu}\Big] + \cos(\nu\varphi)\sin(yD)\Big[\frac{\mathbf{K}_\nu(x)}{x^\nu}\Big]\Big)\Big\},
\end{aligned} \tag{12}$$

which is equivalent to the claim of the theorem. □

The case $\nu = 1$ results in the following formula. To recover a more appropriate result, it suffices to transform (12) in a routine manner.

Corollary 1.1. *For all $z \in \mathbb{C}$, for which $\Re(z) = x > 0, \Im(z) = y \in \mathbb{R}$ we have*

$$\mathbf{K}_1(z) = x\,\cos(yD)\Big[\frac{\mathbf{K}_1(x)}{x}\Big] - y\,\sin(yD)\Big[\frac{\mathbf{K}_1(x)}{x}\Big] + \mathrm{i}\Big\{y\,\cos(yD)\Big[\frac{\mathbf{K}_1(x)}{x}\Big]\Big) + x\,\sin(yD)\Big[\frac{\mathbf{K}_1(x)}{x}\Big]\Big\}.$$

Theorem 2. *For all $\Re(\nu) > -\frac{1}{2}$ and all $z = x + \mathrm{i}y \in \mathbb{C}$, x, y real, we have*

$$\mathbf{M}_\nu(z) = |z|^\nu \left\{ \cos\left(\nu\varphi + yD\Big[\frac{\mathbf{M}_\nu(x)}{x^\nu}\Big]\right) + \mathrm{i}\sin\left(\nu\varphi + yD\Big[\frac{\mathbf{M}_\nu(x)}{x^\nu}\Big]\right)\right\}. \tag{13}$$

Proof. Considering Lemma 2, the derivation of (13) suggests yet another integral representation obtainable from (4), and its derivation:

$$\begin{aligned}
\mathbf{M}_\nu(z) &= \frac{z^\nu}{2^{\nu-1}\sqrt{\pi}\Gamma(\nu+\frac{1}{2})}\int_0^1 \mathrm{e}^{-xt}\big(\cos(yt) - \mathrm{i}\sin(yt)\big)\,(1-t^2)^{\nu-\frac{1}{2}}\,\mathrm{d}t \\
&= \frac{|z|^\nu\big(\cos(\nu\varphi)+\mathrm{i}\sin(\nu\varphi)\big)}{2^{\nu-1}\sqrt{\pi}\Gamma(\nu+\frac{1}{2})}\Big(\mathscr{J}_c^\nu(x,y) - \mathrm{i}\mathscr{J}_s(x,y)\Big) \\
&= |z|^\nu\Big\{\cos(\nu\varphi)\cos(yD)\Big[\frac{\mathbf{M}_\nu(x)}{x^\nu}\Big] + \sin(\nu\varphi)\sin(yD)\Big[\frac{\mathbf{M}_\nu(x)}{x^\nu}\Big] \\
&\quad + \mathrm{i}\Big(\sin(\nu\varphi)\cos(yD)\Big[\frac{\mathbf{M}_\nu(x)}{x^\nu}\Big] + \cos(\nu\varphi)\sin(yD)\Big[\frac{\mathbf{M}_\nu(x)}{x^\nu}\Big]\Big)\Big\}.
\end{aligned} \tag{14}$$

The rest is obvious. □

At the focus of our attention is the special case $\nu = 1$ for the modified Struve function of the second kind, since this generates our motivation for this short note.
Putting $\nu = 1$ in (14) and transforming the variable of the complex term, we establish the following consequence of the Theorem 2.

Corollary 2.1. *For all $z = x + \mathrm{i}y \in \mathbb{C}$, x, y real, we have*

$$\mathbf{M}_1(z) = x\,\cos(yD)\Big[\frac{\mathbf{M}_1(x)}{x}\Big] + y\,\sin(yD)\Big[\frac{\mathbf{M}_1(x)}{x}\Big] + \mathrm{i}\Big(y\,\cos(yD)\Big[\frac{\mathbf{M}_1(x)}{x}\Big] + x\,\sin(yD)\Big[\frac{\mathbf{M}_1(x)}{x}\Big]\Big).$$

4 Discussion

A. Based on the trigonometric–differential operators (5) we conclude the Euler–type formula

$$\mathrm{e}^{\mathrm{i}D}[f(z)] = \cos(D)[f(z)] + \mathrm{i}\sin(D)[f(z)]\,,$$

applied to certain $f \in \mathrm{C}^\infty(\mathbb{R})$ input function. Therefore, we can re–write the statements (11) of Theorem 1 and (13) of Theorem 2 in the form

$$\mathbf{K}_\nu(z) = |z|^\nu\, \mathrm{e}^{\mathrm{i}(\nu\varphi+yD)}\Big[\frac{\mathbf{K}_\nu(x)}{x^\nu}\Big], \qquad \mathbf{M}_\nu(z) = |z|^\nu\, \mathrm{e}^{\mathrm{i}(\nu\varphi+yD)}\Big[\frac{\mathbf{M}_\nu(x)}{x^\nu}\Big].$$

B. The integral representations of the Struve function $\mathbf{H}_\nu(z)$ of the first kind is [6, p. 496, Eq. 12.1.6]

$$\mathbf{H}_\nu(z) = \frac{z^\nu}{2^{\nu-1}\sqrt{\pi}\,\Gamma(\nu+\frac{1}{2})}\int_0^1 \sin(zt)(1-t^2)^{\nu-\frac{1}{2}}\mathrm{d}t, \qquad \Re(\nu) > -\frac{1}{2}\,,$$

and of the modified Struve function $\mathbf{L}_\nu(z)$ of the fist kind [5, p. 292, Eq. 11.5.6] read

$$\mathbf{L}_\nu(z) = \frac{z^\nu}{2^{\nu-1}\sqrt{\pi}\,\Gamma(\nu+\frac{1}{2})}\int_0^{\frac{\pi}{2}} \sinh(z\cos\theta)\sin^{2\nu}\theta\mathrm{d}\theta, \qquad \Re(\nu) > -\frac{1}{2}\,.$$

It is of considerable interest to discuss their structures in the point of view realized above.

C. The idea of application of cosine hyperbolic differential operator is also exploited in the recent article by Górska *et al.* [7, p. 6, Theorem 2.] for presenting the probability density function of the Rice–Middleton model.

5 Conclusion

The model for determining the steady periodic state at the surface of the disk form microelectrode for subsequent determination of the 'approximate' AC impedance for a given simple electron transfer reaction at the surface reads [1–3]

$$\langle \overline{C} \rangle = \frac{2Qw}{\mathbf{D}(p^2+w^2)}\int_0^\infty \frac{J_1^2(ax)}{\sqrt{x^2+q^2}}\frac{\mathrm{d}x}{x}\,,$$

where $\mathbf{D}, Q, w, p$ and $q^2 = p/\mathbf{D}$ are constants in the physical model, whilst $J_1(\cdot)$ stands for the Bessel functions of the first kind of the unit order. After solving the integral and employing appropriate substitutions the mathematical model reduces to finding the imaginary part of the following expression

$$Z = \frac{\mathbf{D}}{a}\Big\{\frac{1}{\sqrt{\mathrm{i}\,t}} + \frac{\mathbf{M}_1(2\sqrt{\mathrm{i}t}\,)}{\mathrm{i}\,t}\Big\}\,.$$

Corollary 2.1. resolves this posed problem. In turn, from the mathematical point of view this short note generalizes the same problem for the real order parameter and complex variable Struve function $\mathbf{K}_\nu(z)$ and the modified Struve function of the second kind $\mathbf{M}_\nu(z)$ as Theorem 2.1. and Theorem 2.2., respectively. Having in mind the point A. in the previous section and the fact that the Euler type formula does not give explicit value for $\mathbf{M}_1(z)$, we explore in the proving procedure the hyperbolic sine and cosine type differentiation operators, getting novel hopefully useful results.

Finally, we point out that the study of integral expressions presented in [8] still remain.

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